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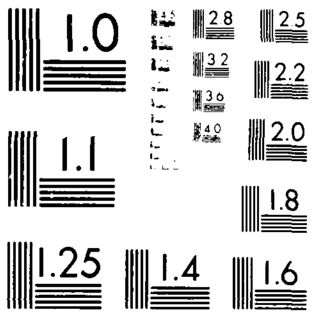

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ROYAL SIGNALS & RADAR ESTABLISHMENT

PLANE WAVE DIFFRACTION BY A THIN DIELECTRIC WEDGE

Author: I D King

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TITLE: PLANE WAVE DIFFRACTION BY A THIN DIELECTRIC WEDGE

AUTHOR: I D King

DATE: May 1987

SUMMARY

The diffraction of a plane electromagnetic wave by a thin dielectric wedge is studied using a set of approximate boundary conditions together with the Wiener-Hopf technique. An integral expression for the scattered field is derived and the diffracted far field obtained by application of the method of steepest descents. The associated GTD diffraction coefficients are exact to order $0(A^2)$, where 2A is the (small) wedge angle. The results are useful for wedges for which $|n^2-1| A \ll 1$, where n is the refractive index of the dielectric relative to that of the surrounding medium.

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PLANE WAVE DIFFRACTION BY A THIN DIELECTRIC WEDGE

I D King

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1 INTRODUCTION

At present there is great interest in the electromagnetic scattering properties of various dielectric and radar absorbent materials (RAM). This information is required, for example, for the calculation of the radar return from dielectric and RAM coated bodies. An important aspect here is the study of diffraction by the edges of such bodies. In the geometrical theory of diffraction (GTD)[1] a wave diffracted by an edge is characterised by a diffraction coefficient. For high frequencies the most significant contribution to the edge diffraction coefficient may be determined from the solution to the problem of the scattering of a plane wave by an infinite wedge[2]. In the absence of a general solution to the problem of plane wave diffraction by an infinite pentrable wedge there have been several approximate formulations. One such formulation is to consider the problem of diffraction by a thin dielectric wedge. Kaminetzky and Keller[2] have derived a set of approximate boundary conditions under the assumption of a small wedge angle. The leading contributions to the diffraction coefficients were then deduced. Analogous boundary conditions have been used by Leppington[3], in a study of travelling waves in thin dielectric slabs, and by Anderson[4] and Chakrabarti[5], in their work on diffraction by a dielectric half-plane.

In the present work, approximate boundary conditions are used in conjunction with the Wiener-Hopf technique to derive an approximate solution for the scattered field external to a thin dielectric wedge. An expression for the diffracted far field is then obtained by the steepest descents technique. The associated diffraction coefficients are exact to order $0(A^2)$, where 2A is the (small) wedge angle. The derivation of these coefficients extends the results contained in reference 2.

In the following section we formulate the problem of the scattering of a plane wave incident normally upon an infinite dielectric wedge. An outline of the derivation of approximate boundary conditions applicable to a wedge of small half-angle is then given. The next two sections deal with the details of the Wiener-Hopf method of solution and with the extraction of expressions for the diffraction coefficients. Finally the limitations of the analysis are discussed and the results are compared with those of other authors and illustrated by numerical examples.

2 FORMULATION OF THE PROBLEM

Consider the electromagnetic scattering of a plane wave incident normally upon an infinite homogeneous dielectric wedge. The wedge, of half-angle A, has permittivity ϵ_1 and permeability μ_0, and is surrounded by another dielectric medium, the latter having permittivity ϵ_0 and the same permeability μ_0. At different times rectangular cartesian coordinates (x,y,z) and circular cylindrical coordinates (r,θ,z) are used, with $x = r\cos\theta$, $y = r\sin\theta$ and $-\pi \leq \theta \leq \pi$. The z-axis is chosen to lie along the edge of the wedge. The interfaces between the two media are at $\theta = \pm(\pi-A)$, as shown in Figure 1. The incident plane wave approaches the wedge from the $\theta = \theta_0$ direction (we assume $0 \leq \theta_0 < \pi-A$) and is assumed time-harmonic. All fields will then exhibit the same time dependence, chosen to be $e^{-i\omega t}$, which will be suppressed throughout.

Without loss of generality we confine the analysis to the cases where the plane wave is either H-polarised or E-polarised. For an H-polarised incident wave the total H-field everywhere has a non-zero z-component only. For example, the external solution is of the form

$$\underline{H} = (0,\ 0,\ H_{zo}(r,\theta)) \tag{1a}$$

$$-i\omega\epsilon_0\underline{E} = \underline{\nabla} \times \underline{H} \quad , \tag{1b}$$

with

$$\left[\nabla_T^2 + k_o^2\right] H_{zo}(r,\theta) = 0 \tag{2}$$

where*

$$\nabla_T^2 \equiv \nabla^2 - \partial_{zz}^2 \qquad \text{and} \qquad k_o^2 = \omega^2\mu_o\epsilon_o \quad .$$

Analogous expressions describe the electromagnetic field within the wedge, where we denote the z-component of the H-field by $H_{z1}(r,\theta)$ and the wavenumber k_0 is replaced by k_1, defined by $k_1^2 = \omega^2\mu_0\epsilon_1$. Continuity of the tangential components of the electric and magnetic fields at the wedge surfaces gives:

$$H_{zo}(\pm\ (\pi-A)) = H_{z1}(\pm\ (\pi-A)) \tag{3a}$$

$$\epsilon_1\partial_\theta H_{zo}(\pm\ (\pi-A)) = \epsilon_o\partial_\theta H_{z1}(\pm\ (\pi-A)) \quad . \tag{3b}$$

Here the r-dependence of the H_{zo} and H_{z1} functions has been suppressed.

Similarly, for an E-polarised incident plane wave the total field external to the wedge is given by

$$\underline{E} = (0,\ 0,\ E_{zo}(r,\theta)) \tag{4a}$$

$$i\omega\mu_o\ \underline{H} = \underline{\nabla} \times \underline{E} \quad , \tag{4b}$$

where E_{zo} satisfies the wave equation

$$\left[\nabla_T^2 + k_o^2\right] E_{zo}(r,\theta) = 0 \quad . \tag{5}$$

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*Throughout the paper, ∂_x denotes a partial derivative with respect to the variable x. The extension to higher order derivatives is obvious.

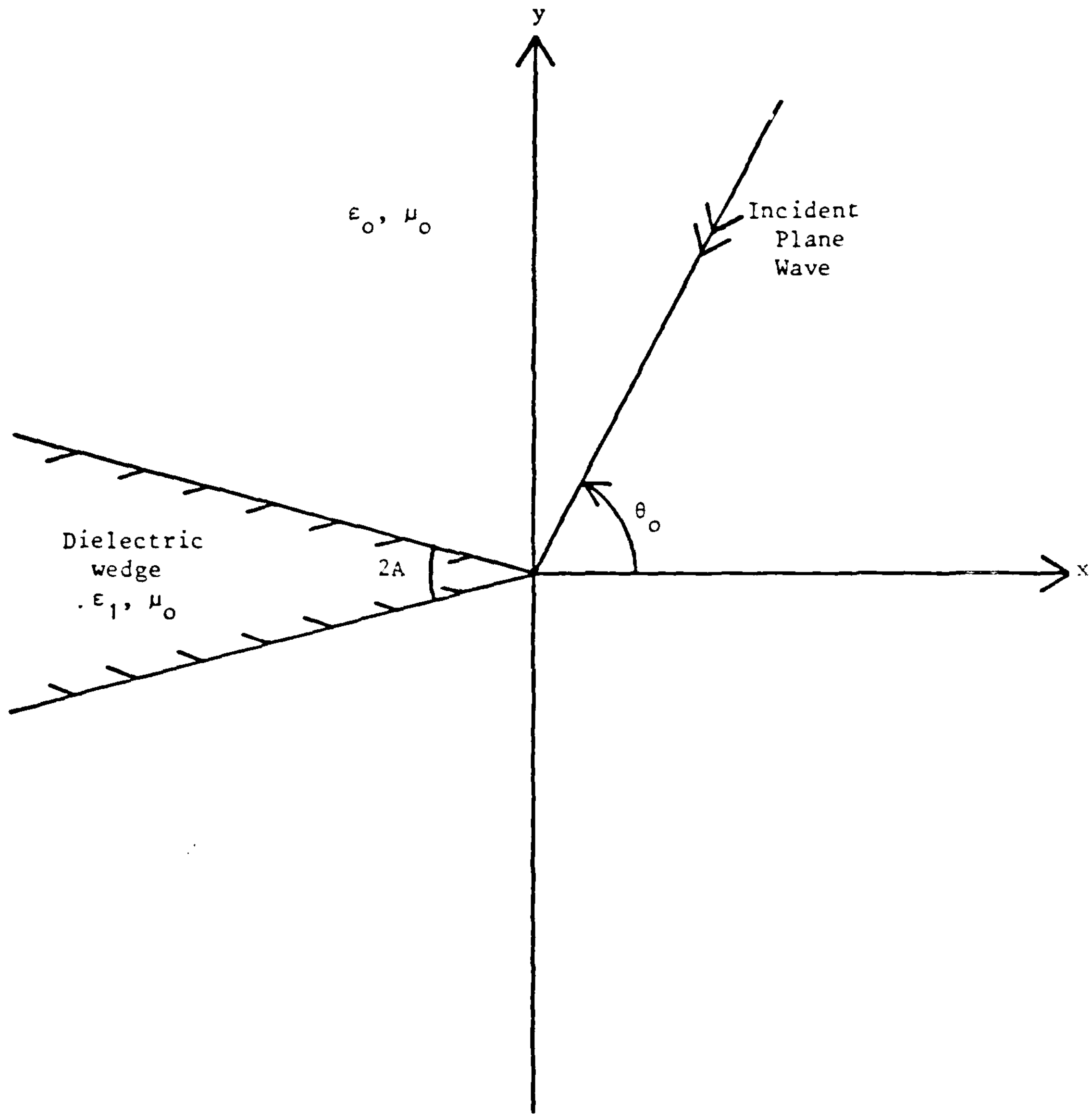

Figure 1. Plane wave incident on a dielectric wedge of half-angle A and relative permittivity $\varepsilon_1/\varepsilon_o$.

In this case the boundary conditions take the form

$$E_{zo}(\pm\ (\pi-A)) = E_{z1}(\pm\ (\pi-A)) \qquad (6a)$$

$$\partial_\theta E_{zo}(\pm\ (\pi-A)) = \partial_\theta E_{z1}(\pm\ (\pi-A)) \quad , \qquad (6b)$$

where E_{z1} is the z-component of the electric field inside the wedge.

To avoid duplication of the subsequent analysis it is convenient to consider scalar fields φ_o and φ_1 defined outside and within the wedge respectively, satisfying the wave equations

$$\left[\nabla_T^2 + k_o^2\right] \varphi_o(r,\theta) = 0 \qquad (7a)$$

and

$$\left[\nabla_T^2 + k_1^2\right] \varphi_1(r,\theta) = 0 \qquad (7b)$$

in their respective domains and the boundary conditions

$$\varphi_o(\pm\ (\pi-A)) = a\ \varphi_1(\pm\ (\pi-A)) \qquad (8a)$$

$$\partial_\theta \varphi_o(\pm\ (\pi-A)) = b\ \partial_\theta \varphi_1(\pm\ (\pi-A)) \quad . \qquad (8b)$$

With an H-polarised incident wave $\varphi \equiv H_z$ and $a = 1$, $b = \epsilon_o/\epsilon_1$. For the E-polarisation case $\varphi \equiv E_z$ and $a = b = 1$. The incident wave is chosen to have unit amplitude. Therefore we write

$$\varphi_o^{inc} = \exp\left[-ik_o(x\cos\theta_o + y\sin\theta_o)\right] \qquad (9)$$

We are primarily concerned with obtaining a solution external to the wedge. In this region we write $\varphi_o = \varphi_o^{inc} + \psi$, so that ψ represents the scattered field and satisfies

$$\left[\nabla_T^2 + k_o^2\right] \psi(r,\theta) = 0 \quad . \qquad (10)$$

At present a general solution to this problem is not available. However, if the wedge half-angle is assumed small ($A << 1$), we can turn to a perturbation analysis.

3. DERIVATION OF APPROXIMATE BOUNDARY CONDITIONS

In this section we derive a set of approximate boundary conditions (abc's) applicable to a study of scattering by a thin dielectric wedge. These conditions take the form of expressions for the discontinuities in the external field and its normal derivative across the half-plane formed by the wedge faces in the limit $A \to 0$.

With $A << 1$, Taylor's theorem may be used inside the wedge to give

$$\varphi_1(\pi-A) - \varphi_1(-\pi+A) = -A(\partial_\theta\varphi_1(\pi-A) + \partial_\theta\varphi_1(-\pi+A)) + O(A^3) \quad (11a)$$

$$\partial_\theta\varphi_1(\pi-A) - \partial_\theta\varphi_1(-\pi+A) = -A(\partial^2_{\theta\theta}\varphi_1(\pi-A) + \partial^2_{\theta\theta}\varphi_1(-\pi+A)) + O(A^3) \quad (11b)$$

Next, the (exact) boundary conditions (8) are used to eliminate all φ_1-dependence from these relations. This gives

$$b[\varphi_o(\pi-A)-\varphi_o(-\pi+A)] = -Aa[\partial_\theta\varphi_o(\pi-A)+\partial_\theta\varphi_o(-\pi+A)]+O(A^3) \tag{12a}$$

$$\begin{aligned} a[\partial_\theta\varphi_o(\pi-A)-\partial_\theta\varphi_o(-\pi+A)] = \; & Ab[r^2\partial^2_{rr}\varphi_o(\pi-A)+r^2\partial^2_{rr}\varphi_o(-\pi+A) \\ & + r\,\partial_r\,\varphi_o(\pi-A)+r\,\partial_r\,\varphi_o(-\pi+A) \\ & + k_1^2\,r^2\varphi_o(\pi-A)+k_1^2r^2\varphi_o(-\pi+A)] + O(A^3) \quad , \end{aligned} \tag{12b}$$

where we have used the wave equation (7b) to rewrite (11b) in a form suitable for the application of exact boundary conditions.

It is unlikely that a solution can be found for φ_o subject to the abc's (12) on the wedge faces. However, φ_1 has been completely eliminated and from now on we need only consider the region external to the wedge. Since the wedge angle is small this region is well approximated by all of three-dimensional space except for the half-plane $y=0$, $x<0$, across which discontinuities in φ_o and $\partial_\theta\varphi_o$ exist. Mathematically, this is justified by further application of Taylor's theorem provided that the domain of φ_o is extended. Using also (7a) the abc's become

$$b[\varphi_o] = -A(a-b)(\partial_\theta\varphi_o(\pi) + \partial_\theta\varphi_o(-\pi)) \tag{13a}$$

and

$$\begin{aligned} a[\partial_\theta\varphi_o] = \; & Ab\left[k_1^2-k_o^2\right]\, r^2(\varphi_o(\pi) + \varphi_o(-\pi)) \\ & + A(a-b)\left[\partial^2_{\theta\theta}\varphi_o(\pi) + \partial^2_{\theta\theta}\varphi_o(-\pi)\right] \quad , \end{aligned} \tag{13b}$$

where

$$[\varphi_o] \equiv \varphi_o(\pi) - \varphi_o(-\pi) \quad , \tag{14a}$$

$$[\partial_\theta\varphi_o] \equiv \partial_\theta\varphi_o(\pi) - \partial_\theta\varphi_o(-\pi) \tag{14b}$$

and the omitted terms are at least of order $O(A^2)$.

In terms of ψ the abc's take the following form (in cartesian coordinates):

$$b(\psi(x,0+) - \psi(x,0-)) + A(a-b)\; x(\partial_y\psi(x,0+) + \partial_y\psi(x,0-)) =$$

$$2A(a-b)\; ik_o x\,\sin\theta_o\, e^{-ik_o x\cos\theta_o} \quad ; \quad x < 0 \tag{15a}$$

$$a(\partial_y\psi(x,0+) - \partial_y\psi(x,0-)) - Ab\left[k_1^2-k_o^2\right] x(\psi(x,0+) + \psi(x,0-))$$

$$- A(a-b)\left[x\partial_{yy}^2\psi(x,0+) - \partial_x\psi(x,0+) + x\partial_{yy}^2\psi(x,0-) - \partial_x\psi(x,0-)\right] =$$

$$2Ab\left[k_1^2-k_o^2\right] x\, e^{-ik_o x\cos\theta_o}$$

$$+ 2A(a-b)\; ik_o(\cos\theta_o + ik_o x\sin^2\theta_o)\, e^{-ik_o x\cos\theta_o} \quad ; \quad x < 0 \quad . \qquad (15b)$$

4 METHOD OF SOLUTION

The problem has been reduced to one of solving the wave equation (10) for the scattered field ψ subject to the abc's (15) and the radiation condition, together with appropriate edge conditions. In addition the following conditions hold on the y=0 plane for $x>0$:

$$\psi(x,0+) = \psi(x,0-) \quad ; \quad x > 0 \qquad (16a)$$

$$\partial_y\psi(x,0+) = \partial_y\psi(x,0-) \quad ; \quad x > 0 \quad . \qquad (16b)$$

The form of the relations (15) and (16) suggests a Wiener-Hopf type approach for solution.

Define the Fourier transform of $\psi(x,y)$ (with respect to the variable x) by

$$\Psi(s,y) = \int_{-\infty}^{\infty} \psi(x,y)\, e^{isx}\, dx = \Psi_+(s,y) + \Psi_-(s,y) \quad , \qquad (17a)$$

where

$$\Psi_+(s,y) = \int_0^{\infty} \psi(x,y)\, e^{isx}\, dx \quad , \qquad (17b)$$

$$\Psi_-(s,y) = \int_{-\infty}^{0} \psi(x,y)\, e^{isx}\, dx \quad . \qquad (17c)$$

Let k_o have a small positive imaginary component ($k_o = k_o^r + ik_o^i$, $0 < k_o^i << k_o^r$) so that the transform Ψ exists and is analytic in the strip $-k_o^i < \text{Im } s < k_o^i \cos\theta_o$.[6] (The assumption of a slightly lossy medium means that Sommerfeld's radiation conditions are equivalent to the requirement that the scattered field vanishes at infinity[7].) Ψ_+ and Ψ_- are analytic in the overlapping half-planes $\text{Im } s > -k_o^i$ and $\text{Im } s < k_o^i \cos\theta_o$ respectively.

Taking the Fourier transform of (10) and solving the resulting differential equation gives

$$\Psi(s,y) = \begin{cases} B_1(s)\ e^{-\gamma(s)y} & ; \quad y > 0 \\ B_2(s)\ e^{\gamma(s)y} & ; \quad y < 0 \end{cases} , \tag{18}$$

with

$$\gamma(s) = \left[s^2 - k_o^2\right]^{\frac{1}{2}} .$$

The s-plane is cut by straight lines from $s = k_o$ to infinity in the upper half-plane and from $s = -k_o$ to infinity in the lower half-plane. The branch of the square root for which $\gamma(o) = -ik_o$ is chosen. Upon transforming equations (16) for $x > 0$ we find

$$\Psi_+(s,0+) = \Psi_+(s,0-) \equiv \Psi_+(s,0) \tag{19a}$$

$$\partial_y\Psi_+(s,0+) = \partial_y\Psi_+(s,0-) \equiv \partial_y\Psi_+(s,0) . \tag{19b}$$

Similarly, transforming equations (15) for $x < 0$, and using (19), gives

$$b(\Psi(s,0+) - \Psi(s,0-)) - iA(a-b)\left[\partial^2_{sy}\Psi(s,0+) + \partial^2_{sy}\Psi(s,0-)\right] + bP_+(s) =$$

$$\frac{2A(a-b)\ ik_o\ \sin\theta_o}{(s-k_o\ \cos\theta_o)^2} \tag{20a}$$

$$a(\partial_y\Psi(s,0+) - \partial_y\Psi(s,0-)) + iAb\left[k_1^2-k_o^2\right](\partial_s\Psi(s,0+) + \partial_s\Psi(s,0-)) +$$

$$iA(a-b)\left[\partial^3_{syy}\Psi(s,0+) - s\Psi(s,0+) + \partial^3_{syy}\Psi(s,0-) - s\Psi(s,0-)\right] + aQ_+(s) =$$

$$\frac{2A[b\left[k_1^2-k_o^2\right] + (a-b)\ k_o(s\cos\theta_o-k_o)]}{(s-k_o\cos\theta_o)^2} , \tag{20b}$$

where

$$bP_+(s) = 2A(a-b)\ i\ \partial^2_{sy}\ \Psi_+(s,0) \tag{21a}$$

$$aQ_+(s) = -2Ab\ i\left[k_1^2-k_o^2\right]\ \partial_s\Psi_+(s,0) -$$

$$2A(a-b)\ i\ (s\ \Psi_+(s,0) + \gamma^2(s)\ \partial_s\Psi_+(s,0)) . \tag{21b}$$

Applying (18), these equations may be rewritten as

$$f(s)\ F_-(s) + P_+(s) + C(s) = 0 \tag{22a}$$

$$g(s)\ G_-(s) + Q_+(s) + D(s) = 0 \quad , \tag{22b}$$

where

$$F_-(s) = B_1(s) - B_2(s) = \Psi_-(s,0+) - \Psi_-(s,0-) \tag{23a}$$

$$G_-(s) = -\gamma(s)(B_1(s) + B_2(s)) = \partial_y\Psi_-(s,0+) - \partial_y\Psi_-(s,0-) \tag{23b}$$

$$f(s) = 1 + iA\,\frac{(a-b)}{b}\left[\frac{s}{\gamma(s)} + \gamma(s)\,\frac{F_-'(s)}{F_-(s)}\right] \tag{24a}$$

$$g(s) = 1 + \frac{iAb}{a}\,\frac{\left(k_1^2-k_o^2\right)}{\gamma(s)}\left[\frac{s}{\gamma^2(s)} - \frac{G_-'(s)}{G_-(s)}\right] - iA\,\frac{(a-b)}{a}\,\gamma(s)\,\frac{G_-'(s)}{G_-(s)} \tag{24b}$$

$$C(s) = -2iA\,\frac{(a-b)}{b}\,\frac{k_o \sin\theta_o}{(s-k_o\cos\theta_o)^2} \tag{25a}$$

$$D(s) = -\frac{2A}{a}\,\frac{[bk_1^2 - ak_o^2 + (a-b)\ sk_o\cos\theta_o]}{(s - k_o\cos\theta_o)^2} \quad , \tag{25b}$$

and the prime denotes the derivative of a function with respect to its argument. (22) are independent Wiener-Hopf equations. Standard techniques may be used to solve equations of this form[8] for the unknown functions F_-, G_-, P_+ and Q_+ (regular in the appropriate half-planes) provided that f, g, C and D are known functions of s, regular in the strip $-k_o^i < \mathrm{Im}\ s < k_o^i \cos\theta_o$. In the current application C and D have been determined, but f and g depend on the unknown functions F_- and G_-.

Suppose for the moment that the Wiener-Hopf equations may be solved for F_- and G_-. Then the scattered field $\psi(x,y)$ may be determined from the Fourier inversion formula

$$\psi(x,y) = -\frac{1}{4\pi}\int_{-\infty+ic}^{\infty+ic} e^{-\gamma|y|-isx}\left[\frac{G_-(s)}{\gamma(s)} - \frac{y}{|y|}\,F_-(s)\right] ds \ , \tag{26}$$

where

$$-k_o^i < c < k_o^i \cos\theta_o \quad .$$

Let us now proceed formally with the method of solution of equations (22) proposed by Jones[8,9]. We assume factorisations of the functions f and g of the form

$$f(s) = f_+(s)\ f_-(s) \tag{27a}$$

$$g(s) = g_+(s)\ g_-(s) \quad . \tag{27b}$$

If, in addition to the analyticity properties of the functions $f_\pm$ and $g_\pm$, we assume that f_+ and g_+ are non-zero in the half-plane Im s > $-k_o^i$ then (22) may be rewritten as

$$f_-(s)\,F_-(s) + \frac{P_+(s)}{f_+(s)} = -\frac{C(s)}{f_+(s)} \equiv -S_+(s) - S_-(s) \qquad (28a)$$

$$g_-(s)\,G_-(s) + \frac{Q_+(s)}{g_+(s)} = -\frac{D(s)}{g_+(s)} \equiv -T_+(s) - T_-(s) \quad , \qquad (28b)$$

where $S_\pm$ and $T_\pm$ are regular in the appropriate half-planes. These decompositions may be carried out in the standard way[10]. The results are

$$S_-(s) = \frac{C(s)}{f_+(k_o\cos\theta_o)}\left[1 - (s-k_o\cos\theta_o)\,\frac{f_+'(k_o\cos\theta_o)}{f_+(k_o\cos\theta_o)}\right] \qquad (29a)$$

$$S_+(s) = \frac{C(s)}{f_+(s)} - S_-(s) \qquad (29b)$$

$$T_-(s) = \frac{D(s)}{g_+(k_o\cos\theta_o)} + \frac{2A}{a}\,\frac{\left[bk_1^2 - ak_o^2 + (a-b)k_o^2\cos^2\theta_o\right]}{(s-k_o\cos\theta_o)}\,\frac{g_+'(k_o\cos\theta_o)}{g_+^2(k_o\cos\theta_o)} \qquad (29c)$$

$$T_+(s) = \frac{D(s)}{g_+(s)} - T_-(s) \qquad (29d)$$

The next step in the procedure is to define functions J(s) and K(s) as

$$J(s) = \frac{P_+(s)}{f_+(s)} + S_+(s) = -f_-(s)\,F_-(s) - S_-(s) \qquad (30a)$$

$$K(s) = \frac{Q_+(s)}{g_+(s)} + T_+(s) = -g_-(s)\,G_-(s) - T_-(s) \qquad (30b)$$

These equations serve to define J(s) and K(s) in the strip $-k_o^i$ < Im s < $k_o^i\cos\theta_o$. We note that the second part of each equation is defined and regular in the region Im s > $-k_o^i$ while the third part is defined and regular in the region Im s < $k_o^i\cos\theta_o$. Since these two half-planes overlap analytic continuation may be used to deduce that the functions J and K are defined and regular in the whole of the complex s-plane. If, in addition, J(s) and K(s) tend to zero (and have algebraic behaviour) as s tends to infinity in all directions it follows from an extended form of Liouville's theorem[11] that these functions are identically zero. Such conditions on the asymptotic behaviour of J and K are necessary to ensure a unique solution to the problem. A unique solution will be obtained once suitable conditions on the behaviour of ψ at the edge x = y = 0 have been imposed.

To specify a sufficient set of edge conditions we must determine the behaviour of the quantities in the second and third parts of (30) as s tends to infinity in the appropriate half-planes. Let us suppose that

$$\psi(x,y) \sim O(x^{\mu}) \text{ as } x \to 0+ \text{ on } y = 0 \tag{31a}$$

$$\psi(x,y) \sim O(x^{\nu}) \text{ as } x \to 0- \text{ on } y = 0\pm \tag{31b}$$

$$\psi_y(x,y) \sim O(x^{\rho}) \text{ as } x \to 0+ \text{ on } y = 0 \tag{31c}$$

$$\psi_y(x,y) \sim O(x^{\sigma}) \text{ as } x \to 0- \text{ on } y = 0\pm \tag{31d}$$

It follows from Abelian theorems on Fourier transforms that

$$F_-(s) \sim O(s^{-\nu-1}) \text{ as } s \to \infty \text{ in } \operatorname{Im} s < k_o^i \cos\theta_o \tag{32a}$$

$$G_-(s) \sim O(s^{-\sigma-1}) \text{ as } s \to \infty \text{ in } \operatorname{Im} s < k_o^i \cos\theta_o \tag{32b}$$

$$P_+(s) \sim O(s^{-\rho-2}) \text{ as } s \to \infty \text{ in } \operatorname{Im} s > -k_o^i \tag{32c}$$

$$Q_+(s) \sim O(s^{-\mu}) \quad \text{ as } s \to \infty \text{ in } \operatorname{Im} s > -k_o^i \quad . \tag{32d}$$

From (25), (29a) and (29c) is is clear that

$$S_-(s) \to 0 \text{ as } s \to \infty \text{ in } \operatorname{Im} s < k_o^i \cos\theta_o \tag{33a}$$

$$T_-(s) \to 0 \text{ as } s \to \infty \text{ in } \operatorname{Im} s < k_o^i \cos\theta_o \quad . \tag{33b}$$

It remains to determine the asymptotic behaviour of $f_\pm(s)$ and $g_\pm(s)$. From (24a), (24b), (32a) and (32b) we find that

$$f(s) \to 1 + iAc_1 \operatorname{sgn}(s) \text{ as } |s| \to \infty \tag{34a}$$

$$g(s) \to 1 + iAc_2 \operatorname{sgn}(s) \text{ as } |s| \to \infty \quad . \tag{34b}$$

where the constants c_i $(i = 1,2)$ are given by

$$c_1 = -\frac{\nu(a-b)}{b} \tag{35a}$$

$$c_2 = \frac{(\sigma+1)(a-b)}{a} \tag{35b}$$

and

$$\operatorname{sgn}(s) = \pm 1 \text{ for } \operatorname{Re} s \gtrless 0 \quad . \tag{36}$$

The required factorisations may be performed approximately if we assume that $A|c_i| << 1$. By using the sum split[3]

$$\mathrm{sgn}(s) = 1 + \frac{i}{\pi}\, \ell n_+ \hat{s} - \frac{i}{\pi}\, \ell n_- \hat{s} \quad , \tag{37}$$

where the dimensionless variable $\hat{s}$ is chosen to be $\hat{s} = s/|s|$ and $\ell n_+ s$ and $\ell n_- s$ have branch cuts in the lower and upper half-planes respectively and are real when s is real and positive, it follows that*

$$1+iAc_i\,\mathrm{sgn}(s) = \{d_i(1+iAc_i(e_i + \frac{i}{\pi}\,\ell n_+\hat{s}))\}\{d_i^{-1}(1+iAc_i(1-e_i - \frac{i}{\pi}\,\ell n_-\hat{s}))\} +$$

$$0\left[|Ac_i|^2\right] \quad ; \quad i = 1,\ 2 \tag{38}$$

(d_1, d_2, e_1 and e_2 are arbitrary constants.) The quantity in the first (second) set of squared brackets is regular as $s \to \infty$ in the half-plane Im $s > -k_0^i$ (Im $s < k_0^i \cos\theta_0$). We conclude that

$$f_+(s) \text{ and } g_+(s) \to \text{constant as } s \to \infty \text{ in Im } s > -k_o^i \tag{39a}$$

$$f_-(s) \text{ and } g_-(s) \to \text{constant as } s \to \infty \text{ in Im } s < k_o^i \cos\theta_o . \tag{39b}$$

From (29b) and (29d) it now follows that

$$S_+(s) \text{ and } T_+(s) \to 0 \text{ as } s \to \infty \text{ in Ims} > -k_o^i \quad . \tag{40}$$

In accordance with our earlier remarks we see from (30a) that for J(s) to vanish everywhere on the complex s-plane it is sufficient to assume that either $\rho > -2$ or $\nu > -1$. Similarly, for K(s) to vanish identically it is sufficient that either $\mu > 0$ or $\sigma > -1$. These conditions are satisfied if, for example, we require that $\psi_y(x,y)$ possesses at most an integrable singularity at the point $x = y = 0$.

For the moment we assume a unique solution to the problem so that J(s) and K(s) are zero for all complex s. The scattered field is then given by

$$\psi(x,y) = \frac{1}{4\pi}\int_{-\infty+ic}^{\infty+ic} e^{-\gamma(s)|y|-isx}\left[\frac{T_-(s)}{\gamma(s)\,g_-(s)} - \frac{y}{|y|}\,\frac{S_-(s)}{f_-(s)}\right] ds \tag{41}$$

where we have assumed that $f_-(s)$ and $g_-(s)$ are non-zero in the half-plane Im $s < k_0^i \cos\theta_0$. With the known asymptotic dependence of the functions $S_-(s)$, $T_-(s)$, $f_-(s)$ and $g_-(s)$, it is a straightforward matter[12] to deduce that $\mu \geq 1$ and $\nu \geq 0$ for this solution. This behaviour confirms that the set of sufficient edge conditions derived above is indeed satisfied. Also, it ensures that the component of the total field parallel to the wedge (ie φ_0) is finite at the edge, a necessary condition on the solution since it is a consequence of the requirement that the electromagnetic energy in any finite domain be finite[13].

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*If, for example, we choose to define $\hat{s} = s/k_0^r$ then the terms omitted from the right side of (38) are small only for $|s| \approx k_0^r$.

From (41) we see that factorisations of f(s) and g(s) are required for a determination of ψ. Since J(s) and K(s) are now known to vanish identically, (30) may be used to rewrite (24a) and (24b) as

$$f_+(s)f_-(s) = 1 + iA\,\frac{(a-b)}{b}\left\{\frac{s}{\gamma(s)} - \gamma(s)\,\frac{f_-'(s)}{f_-(s)}\right.$$

$$\left. - \frac{2\gamma(s)}{(s-k_o\cos\theta_o)}\,\frac{\left[1 - \frac{1}{2}(s-k_o\cos\theta_o)\,\dfrac{f_+'(k_o\cos\theta_o)}{f_+(k_o\cos\theta_o)}\right]}{\left[1 - (s-k_o\cos\theta_o)\,\dfrac{f_+'(k_o\cos\theta_o)}{f_+(k_o\cos\theta_o)}\right]}\right\} \qquad (42a)$$

$$g_+(s)g_-(s) = 1 + iA\,\frac{b}{a}\,\frac{\left[k_1^2-k_o^2\right]s}{\gamma^3(s)} + \frac{iA}{a}\left[\frac{b\left[k_1^2-k_o^2\right]}{\gamma(s)}\right.$$

$$\left. + (a-b)\gamma(s)\right]\left\{\frac{g_-'(s)}{g_-(s)} + \frac{\left[2\bar{k}+s\cos\theta_o + k_o\cos^2\theta_o\right]}{\left[\bar{k}+s\cos\theta_o\right]\left[s-k_o\cos\theta_o\right]}\right.$$

$$\left.\frac{\left[1 - \dfrac{\left[\bar{k}+k_o\cos^2\theta_o\right]\left[s-k_o\cos\theta_o\right]}{\left[2\bar{k} + s\cos\theta_o + k_o\cos^2\theta_o\right]}\,\dfrac{g_+'(k_o\cos\theta_o)}{g_+(k_o\cos\theta_o)}\right]}{\left[1 - \dfrac{\left[\bar{k}+k_o\cos^2\theta_o\right]\left[s-k_o\cos\theta_o\right]}{\left[\bar{k}+s\cos\theta_o\right]}\,\dfrac{g_+'(k_o\cos\theta_o)}{g_+(k_o\cos\theta_o)}\right]}\right\}, \qquad (42b)$$

where

$$\bar{k} = \frac{bk_1^2 - ak_o^2}{(a-b)k_o} \qquad (43)$$

To proceed further, it appears that complicated 'transcendental' factorisations must be found; ie the functions to be factorised depend upon the results of the factorisations! In fact the situation is not so serious. If we restrict the analysis to those thin-wedge ($A << 1$) problems where the electromagnetic parameters are such that

$$A\left|\frac{a-b}{b}\right| << 1 \qquad A\left|\frac{a-b}{a}\right| << 1 \qquad A\left|\frac{b}{a}\,\frac{\left[k_1^2-k_o^2\right]}{k_o^2}\right| << 1 \qquad (44)$$

then approximate factorisations are possible. When these conditions are satisfied it is meaningful to look for solutions $f_\pm$ and $g_\pm$ of the form*

$$f_\pm(s) = f_1^{\pm 1}\left\{1 + iA\,\frac{(a-b)}{b}\left[\hat{F}_\pm(s) \pm f_2\right] + O(A^2)\right\} \qquad (45a)$$

$$g_\pm(s) = g_1^{\pm 1}\left\{1 + iA\,\frac{(a-b)}{a}\left[\hat{G}_\pm^{(1)}(s) \pm g_2\right]\right.$$

$$\left. + iA\,\frac{b}{a}\,\frac{\left[k_1^2-k_o^2\right]}{k_o^2}\left[\hat{G}_\pm^{(2)}(s) \pm g_3\right] + O(A^2)\right\}$$

$$\equiv g_1^{\pm 1}\left\{1 + iA(\hat{G}_\pm(s) \pm g_4) + O(A^2)\right\} \qquad , \qquad (45b)$$

where $f_{1,2}$ and $g_{1,2,3,4}$ are arbitrary finite constants. This leads to two simplifications. First, the functions to be factorised may be expanded in powers of the small parameters (44). We choose to neglect those terms that are at least of order $O(A^2)$. Second, the factorisations are reduced to the easier problem of performing sum decompositions, with the errors again of order $0(A^2)$. The sum splits to be found are

$$\hat{F}(s) = \hat{F}_+(s) + \hat{F}_-(s) = \frac{s}{\gamma(s)} - \frac{2\gamma(s)}{(s - k_o\cos\theta_o)} \qquad (46a)$$

$$\hat{G}(s) = \hat{G}_+(s) + \hat{G}_-(s) = \frac{(a-b)}{a}\cdot\frac{2\gamma(s)}{(s - k_o\cos\theta_o)}$$

$$+ \frac{b}{a}\left[k_1^2-k_o^2\right]\left[\frac{s}{\gamma^3(s)} + \frac{2}{\gamma(s)(s - k_o\cos\theta_o)}\right]$$

$$- \frac{\cos\theta_o}{\left[s\cos\theta_o + \bar{k}\right]}\left[\frac{(a-b)}{a}\,\gamma(s) + \frac{b}{a}\,\frac{\left[k_1^2-k_o^2\right]}{\gamma(s)}\right] \quad . \qquad (46b)$$

Thus, when the functions $\hat{F}_\pm$ and $\hat{G}_\pm$ have been determined, the factorisations (42) will be known, correct to first order in the parameters (44).

The decomposition (46a) is carried out in the usual manner. We find

*These are the most general forms consistent with the known behaviour (39) of the functions as $|s| \to \infty$ in the appropriate half-planes.

Note also that provided we choose the parameters (44) small enough we can validate our earlier assumptions on the non-vanishing of $f_\pm$ and $g_\pm$ in the appropriate half-planes.

$$\hat{F}_{\pm}(s) = \mp \frac{i}{\pi(s-k_o\cos\theta_o)} \left\{ 2k_o\theta_o\sin\theta_o + \frac{\left[2k_o^2 - s(s+k_o\cos\theta_o)\right]}{\left[k_o^2-s^2\right]^{\frac{1}{2}}} \right.$$

$$\left. \left\{ \tan^{-1} \frac{s}{\left[k_o^2-s^2\right]^{\frac{1}{2}}} \mp \frac{\pi}{2} \right\} \right\} \qquad (47)$$

when $|s| < |k_o|$.

Although it appears that the function $\hat{G}(s)$ is singular at the point

$$s = -\frac{\bar{k}}{\cos\theta_o}$$

this is not so for either set of boundary conditions of physical interest. When $a = 1$ and $b = \epsilon_2\epsilon_1$ (H-polarised incident plane wave), $\bar{k}$ vanishes and the singularity is removable. Also, when $a = b = 1$ (E-polarised incident wave), $|\bar{k}|$ becomes infinite and the final term on the right side of (46b) is absent. The result of the sum split, again valid for $|s| < |k_o|$, is as follows:

$$\hat{G}_{\pm}(s) = \pm \frac{(a-b)}{a} \frac{i}{\pi} \left[\frac{2}{s-k_o\cos\theta_o} - \frac{\cos\theta_o}{s\cos\theta_o + \bar{k}} \right] \left[k_o^2 - s^2\right]^{\frac{1}{2}}$$

$$\left\{ \tan^{-1} \frac{s}{\left[k_o^2-s^2\right]^{\frac{1}{2}}} \mp \frac{\pi}{2} \right\}$$

$$\pm \frac{(a-b)}{a} \frac{2ik_o\theta_o\sin\theta_o}{\pi(s-k_o\cos\theta_o)}$$

$$\mp \frac{b}{a} \left[k_1^2 - k_o^2\right] \frac{i}{\pi} \left\{ \frac{2}{s-k_o\cos\theta_o} - \frac{\cos\theta_o}{s\cos\theta + \bar{k}} - \frac{s}{k_o^2-s^2} \right\} \left[k_o^2-s^2\right]^{-\frac{1}{2}}$$

$$\left\{ \tan^{-1} \frac{s}{\left[k_o^2-s^2\right]^{\frac{1}{2}}} \mp \frac{\pi}{2} \right\}$$

$$\pm \frac{b}{a} \left[k_1^2 - k_o^2\right] \frac{i}{\pi} \left\{ \frac{1}{k_o^2 - s^2} - \frac{2\theta_o}{k_o\sin\theta_o(s-k_o\cos\theta_o)} \right\} \qquad (48)$$

In terms of these split functions the scattered field is

$$\psi(x,y) = \int_{-\alpha+ic}^{\alpha+ic} w(s)\, e^{-\gamma(s)|y| - isx}\, ds \quad , \tag{49}$$

with

$$w(s) = -\frac{A(a-b)}{a}\frac{k_o}{2\pi}\frac{1}{(s-k_o\cos\theta_o)^2\,\gamma(s)}\Big\{(s\cos\theta_o + \bar{k})$$

$$\cdot\,[1 - iA(\hat{C}_-(s) + \hat{C}_+(k_o\cos\theta_o))]$$

$$-\left[\bar{k} + k_o\cos^2\theta_o\right](s - k_o\cos\theta_0)\; iA\;\hat{C}'_+(k_o\cos\theta_o)$$

$$-\,i\frac{y}{|y|}\frac{a}{b}\sin\theta_o\,\gamma(s)\left[1 - \frac{iA(a-b)}{b}\right.$$

$$\left.\left.\cdot\,(\hat{F}_-(s) + \hat{F}_+(k_o\cos\theta_o) + (s - k_o\cos\theta_o)\,\hat{F}'_+(k_o\cos\theta_o))\right]\right\} . \tag{50}$$

Again, expansions in the small parameters (44) have been used to arrive at this form. This result gives exact expressions for the first two terms* of an expansion of ψ in powers of the wedge angle A. In particular, note that to this order all dependence on the arbitrary constants $f_{1,2}$ and $g_{1,2,3,4}$ disappears.

5 DERIVATION OF EDGE DIFFRACTION COEFFICIENTS

Contour integrals of the type (49) arise frequently in the treatment of problems to which the Wiener-Hopf technique may be applied. Noble[14] has given a useful account of the analysis of this integral.

At this stage it is convenient to let k_o^i tend to zero, so that the integration contour lies along the real axis of the s-plane, with indentations as shown in Figure 2a. It is standard procedure to revert to polar coordinates and to make the substitution $s = -k_o\cos\beta$. Under this transformation the s-plane is mapped into an infinite number of parallel strips of the β-plane. Choosing the segment $0 \le \mathrm{Re}\beta \le \pi$ the integration contour becomes the contour C of Figure 2b. The integral becomes

$$\psi(r,\theta) = k_o\int_C w(-k_o\cos\beta)\, e^{ik_o r\cos(\beta\mp\theta)}\sin\beta\, d\beta \quad , \tag{51}$$

where the upper (lower) sign applies for $0 \le \theta \le \pi$ $(-\pi \le \theta \le 0)$.

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*It is easily verified that the inclusion of terms of order $0(A^2)$ in the abc's (13) leads to corrections to the scattered field which are at least of order $0(A^3)$.

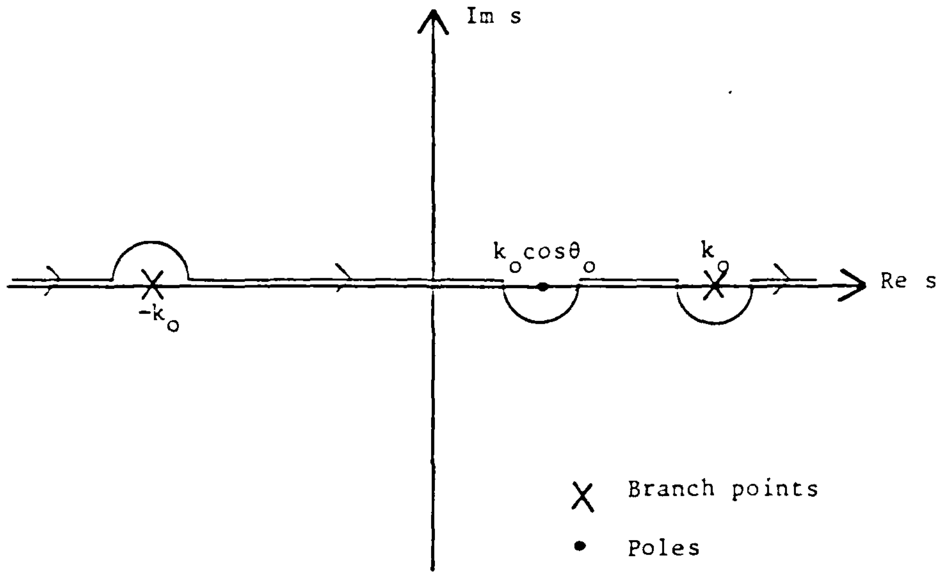

Figure 2(a). Integration contour in the s-plane.

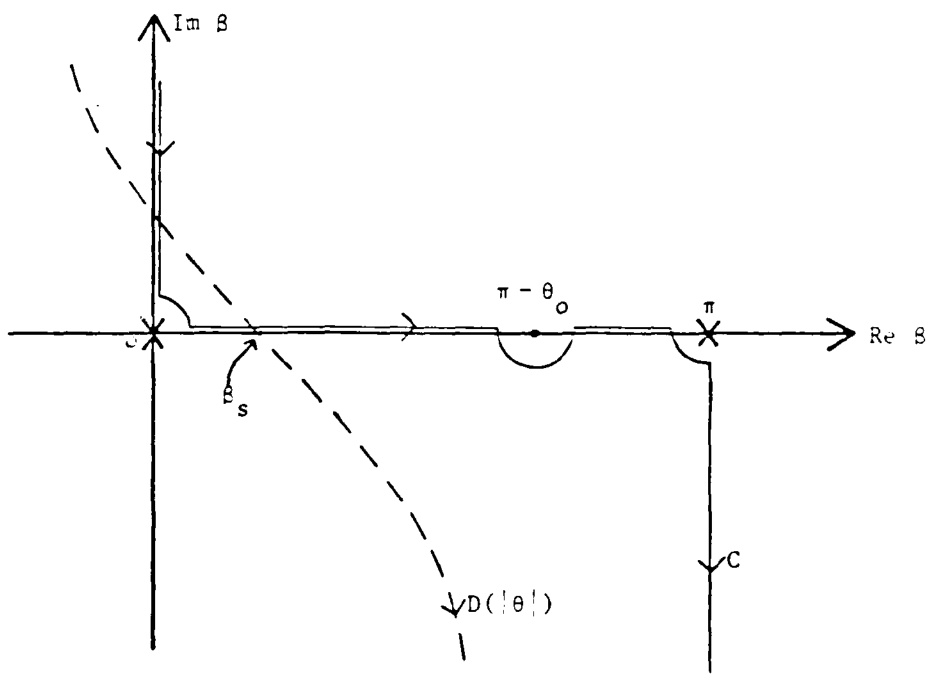

Figure 2(b). Integration contour C and steepest descents path D(|θ|) in the β-plane.

An approximate evaluation of (51) for large values of the parameter $k_o r$ may be obtained by the method of steepest descents. The saddle point of the integrand is at $\beta = \beta_s = |\theta|$ and the path of steepest descent is $D(|\theta|)$. (See Figure 2b.) In the deformation of the path C into the new contour $D(|\theta|)$, the pole at $\beta = \pi - \theta_o$ is captured when $|\theta| > \pi - \theta_o$. For $0 \leq \theta \leq \pi$, the threshold occurs at $\theta = \pi - \theta_o$ and corresponds to the reflection boundary of the half-plane $y = 0$, $x < 0$. Thus the contribution of the pole is associated with the reflected wave. Similarly, when $-\pi \leq \theta \leq 0$, the pole term is included only in the shadow sector, and gives rise to the field transmitted through the dielectric. The integral along the steepest descent path $D(|\theta|)$ is interpreted as the diffracted field $\psi_d(r,\theta)$. Thus

$$\psi(r,\theta) = \psi_d(r,\theta) + mk_o \int_{C_p} w(-k_o\cos\beta)\, e^{ik_o r\cos(\beta-|\theta|)} \sin\beta\, d\beta \quad , \qquad (52)$$

with

$$\psi_d(r,\theta) = k_o \int_{D(|\theta|)} w(-k_o\cos\beta)\, e^{ik_o r\cos(\beta-|\theta|)} \sin\beta\, d\beta \qquad (53)$$

and

$$m = \begin{cases} 0 & , \quad |\theta| < \pi - \theta_o \\ 1 & , \quad |\theta| > \pi - \theta_o \end{cases} \qquad (54)$$

The contour C_p encloses the pole at $\beta = \pi - \theta_o$ (and no other singularities) and is traversed in the positive (anti-clockwise) sense.

Since we are primarily concerned with a study of diffraction we shall consider ψ_d only. An asymptotic expansion of the type suggested by Jones[15] may be used to evaluate ψ_d for $k_o r >> 1$. However, for the purposes of extracting the diffraction coefficient, it is sufficient to determine the diffracted far field, which is obtained from the leading term of such an expansion. Upon inspection of (50) it is apparent that this procedure breaks down in the vicinity of the reflection and shadow boundaries; ie when the pole of the integrand at $\beta = \pi - \theta_o$ is close to the saddle point at $\beta = |\theta|$.

Thus, by the method of steepest descents, it is readily shown that

$$\psi_d(r,\theta) \xrightarrow{k_o r \to \infty} \sqrt{2\pi}\, k_o\, |\sin\theta|\, w(-k_o\cos\theta)\, \frac{e^{i(k_o r - \pi/4)}}{\sqrt{k_o r}} \quad , \qquad (55)$$

provided that $\cos\theta + \cos\theta_o \neq 0$.

As expected, in the far field ψ_d appears as a cylindrical wave originating at the diffracting edge. The diffraction coefficient $D(\theta,\theta_o)$ is defined by the relation

$$\psi_d(r,\theta) \xrightarrow{k_o r \to \infty} D(\theta, \theta_o) \cdot \frac{e^{ik_o r}}{\sqrt{k_o r}} \quad . \qquad (56)$$

From (50), (55) and (56), then, we deduce that

$$D(\theta,\theta_o) = -A\,\frac{(a-b)}{a}\,\frac{e^{i\pi/4}}{\sqrt{2\pi}}\,\frac{1}{\left[\cos\theta+\cos\theta_o\right]^2}\left\{\frac{\left[\bar{k}-k_o\cos\theta\cos\theta_o\right]}{k_o}\right.$$

$$\cdot\left[1 - iA(\hat{C}_-(-k_o\cos\theta) + \hat{C}_+(k_o\cos\theta_o))\right]$$

$$+ iA\left[\bar{k}+k_o\cos^2\theta_o\right](\cos\theta+\cos\theta_o)\;\hat{C}_+'(k_o\cos\theta_o)$$

$$-\frac{a}{b}\sin\theta\,\sin\theta_o\left[1 - \frac{iA(a-b)}{b}\left[\hat{F}_-(-k_o\cos\theta)\, +\right.\right.$$

$$\left.\left.\left.+\hat{F}_+(k_o\cos\theta_o) - k_o(\cos\theta+\cos\theta_o)\;\hat{F}_+'(k_o\cos\theta_o)\right]\right]\right\} \tag{57}$$

for $0 \leq \theta_o < \pi-A$ and $\cos\theta + \cos\theta_o \neq 0$, provided that the strong inequalities (44) are satisfied.

It is clear that ψ_d is invariant under the simultaneous replacements $\theta \to -\theta$ and $\theta_o \to -\theta_o$. This is consistent with the mirror symmetry of the diffracting object in the plane $y = 0$ and so we deduce that the expression (57) obtained for the diffraction coefficient is valid also for $-(\pi-A) < \theta_o \leq 0$.

From the reciprocity of Maxwell's equations it follows that the diffraction coefficient should be unaltered by the interchange of incident angle θ_o and diffraction angle θ. Indeed, this invariance should apply to each term of an expansion of the diffraction coefficient in powers of wedge angle A. It remains to rewrite (57) in a form which makes this symmetry evident. By using the explicit forms (47) and (48) for the functions $F_\pm(s)$ and $G_\pm^{(1,2)}(s)$, it is found that

$$D(\theta,\theta_o) = \frac{e^{i\pi/4}}{\sqrt{2\pi}}\cdot\frac{1}{\left[\cos\theta+\cos\theta_o\right]^2}\left\{A\,\frac{(a-b)}{a}\,\frac{(k_o\cos\theta\cos\theta_o-\bar{k})}{k_o}\right.$$

$$\left[1 - \frac{A}{\pi}\,\frac{(a-b)}{a}\left[\Gamma\left[\theta,\theta_o\right] + \Gamma\left[\theta_o,\theta\right]\right]\right]$$

$$+ A\,\frac{(a-b)}{b}\sin\theta\,\sin\theta_o$$

$$\left.\cdot\left[1 - \frac{A}{\pi}\,\frac{(a-b)}{b}\,(\Phi(\theta,\theta_o) + \Phi(\theta_o,\theta))\right]\right\}\;, \tag{58}$$

with

$$\Gamma\left[\theta_1,\theta_2\right] = \frac{\left[\bar{k}+k_o\cos^2\theta_1\right]}{k_o\sin^2\theta_1}\left[1+\frac{2\theta_1\sin\theta_1}{(\cos\theta_1+\cos\theta_2)}+\frac{k_o\theta_1\sin\theta_1\cos\theta_2}{\left[\bar{k}-k_o\cos\theta_1\cos\theta_2\right]}\right]$$

$$-\frac{\theta_1\cot\theta_1\left[\bar{k}+k_o\right]}{k_o\sin^2\theta_1} \tag{59}$$

and

$$\Phi(\theta_1,\theta_2) = 1+\frac{\theta_1\left[1+\sin^2\theta_1+\cos\theta_1\cos\theta_2\right]}{\sin\theta_1(\cos\theta_1+\cos\theta_2)} \quad . \tag{60}$$

It is easily checked that, once the differences in notation have been reconciled, the leading terms of (58) (ie those linear in wedge angle A) reproduce the results obtained by Kaminetzky and Keller[2].

6 DISCUSSION OF RESULTS

We begin by summarising our results. When the incident plane wave is H-polarised the GTD diffraction coefficient is

$$D^H(\theta,\theta_o) = \frac{e^{i\pi/4}}{\sqrt{2\pi}}\,\frac{1}{\left[\cos\theta+\cos\theta_o\right]^2}\left\{A\,\frac{(n^2-1)}{n^2}\,\cos\theta\cos\theta_o\right.$$

$$\left[1-\frac{A}{\pi}\,\frac{(n^2-1)}{n^2}\left[\Gamma^H(\theta,\theta_o)+\Gamma^H(\theta_o,\theta)\right]\right]$$

$$+A(n^2-1)\sin\theta\sin\theta_o$$

$$\left.\left[1-\frac{A}{\pi}(n^2-1)(\Phi(\theta,\theta_o)+\Phi(\theta_o,\theta))\right]\right\} \quad , \tag{61}$$

where

$$\Gamma^H(\theta_1,\theta_2) = \cot^2\theta_1\left[1-\frac{2\theta_1\tan\theta_1\cos\theta_2}{(\cos\theta_1+\cos\theta_2)}-\theta_1\cot\theta_1\right] \tag{62}$$

$\Phi(\theta_1,\theta_2)$ is given by (60) and $n = k_1/k_o$ is the refractive index of the dielectric relative to the surrounding medium (taken to be air).

For an E-polarised incident wave the diffraction coefficient is

$$D^E(\theta,\theta_o) = -\frac{e^{i\pi/4}}{\sqrt{2\pi}} \frac{A(n^2-1)}{\left[\cos\theta+\cos\theta_o\right]^2}$$

$$\cdot \left[1 - \frac{A}{\pi}(n^2-1)(\Gamma^E(\theta,\theta_o) + \Gamma^E(\theta_o,\theta))\right] \quad , \qquad (63)$$

with

$$\Gamma^E(\theta_1,\theta_2) = \operatorname{cosec}^2\theta_1 \left[1 + \frac{2\theta_1\sin\theta_1}{(\cos\theta_1+\cos\theta_2)} - \theta_1\cot\theta_1\right] \quad . \qquad (64)$$

In both cases the diffracted far-field is given by (56). These expressions are valid only when the order $O(A^2)$ corrections are small in comparison with the leading contributions. This requirement places a number of restrictions on the permissible values of the various angles and parameters:

(i) Assuming that $n^2 > 1$, (61) and (63) are useful only when $(n^2-1)\ A << 1$.

(ii) We have already noted the breakdown of the analysis in the vicinity of the reflection and shadow boundaries; ie when $\theta_0 \approx \pi \mp \theta$.

(iii) We observe that the functions Γ^H, Γ^E and $\ddagger$ are all singular as $\theta_1 \to \mp \pi$, implying that the formulae (61) and (63) are invalid when either the incident or diffracted wave approaches one of the faces of the thin wedge.

It is worth pointing out that although equations (61) and (63) show the diffraction coefficients as expansions in powers of

$$\frac{(n^2-1)}{n^2} A \quad \text{and} \quad (n^2-1)\ A \quad ,$$

there is no guarantee that such a pattern will persist to higher orders. The missing terms are considered negligible only because of the assumption that $A << 1$.

The results (61) and (63) remain valid for a lossy dielectric provided that the condition $|n^2-1|\ A << 1$ is not violated. Therefore the results of this analysis cannot be applied to a study of scattering from a thin wedge with high conductivity. In particular, a reconciliation with the known diffraction coefficients of a perfectly conducting wedge is not possible.*

*Of course, a similar Wiener-Hopf type approach with suitable approximate boundary conditions may be used in a study of scattering by a thin perfectly conducting wedge. The essential difference is that in this case the diffraction coefficients remain non-zero as A becomes vanishingly small, since then the diffracting object is a perfectly conducting half-plane.

Kaminetzky and Keller[2] have used a perturbation approach to deduce diffraction coefficients for nearly transparent (diaphanous) wedges. Specifically, expressions linear in the small parameter ϵ, which is a measure of the difference of the refractive index of the dielectric from that of the surrounding medium, have been obtained. Furthermore it was verified in Ref 2 that both this type of analysis and a perturbative study of thin wedges give identical leading terms, ie terms of order $0(\epsilon A)$, for diffraction coefficients of wedges which are both thin and almost transparent. It is a simple task to use the results of the present work to extend this check to include a comparison of contributions of order $0(\epsilon A^2)$. In fact, it is found that in both types of expansion there are no such terms.

It has already been remarked that the leading contributions to the thin wedge dielectric-edge diffraction coefficients (61) and (63) are in accord with the earlier work of Kaminetzky and Keller. The effect of including the second order terms is illustrated in Figure 3, where the magnitudes of the diffraction coefficients for a normally incident plane wave are plotted as functions of wedge angle. The corrections are more significant for the case of E-polarisation, where they are of opposite sign to the leading contributions.

In Figure 4 the behaviour of the diffraction coefficients is shown as a function of observation angle for various angles of incidence.

It appears that there is no available experimental data on the scattering of electromagnetic radiation by penetrable wedges. Yeo, Wall and Bates[16,17] have embarked on a programme of work aimed at developing an algorithm for the computation of dielectric-edge diffraction coefficients. The results obtained for small wedge angles using this combined analytical-computational approach have been compared with those of Ref 2 and show good agreement.

7 CONCLUSIONS

A detailed analysis of plane wave diffraction by a thin dielectric wedge has been given. Particular attention has been given to the role of edge conditions and their relevance to the question of uniqueness of the solution. The solution for the scattered field has been obtained in the form of a contour integral. The fields reflected and transmitted by the wedge are associated with the pole terms. In the far field region the remainder of the solution appears to emanate from the edge of the wedge and is interpreted as being due to diffraction by the edge. The corresponding GTD edge diffraction coefficients are obtained as expansions in the small wedge angle A, but are valid only when $|n^2-1|\, A \ll 1$. The results comply with the requirements of reciprocity and are exact to order $0(A^2)$.

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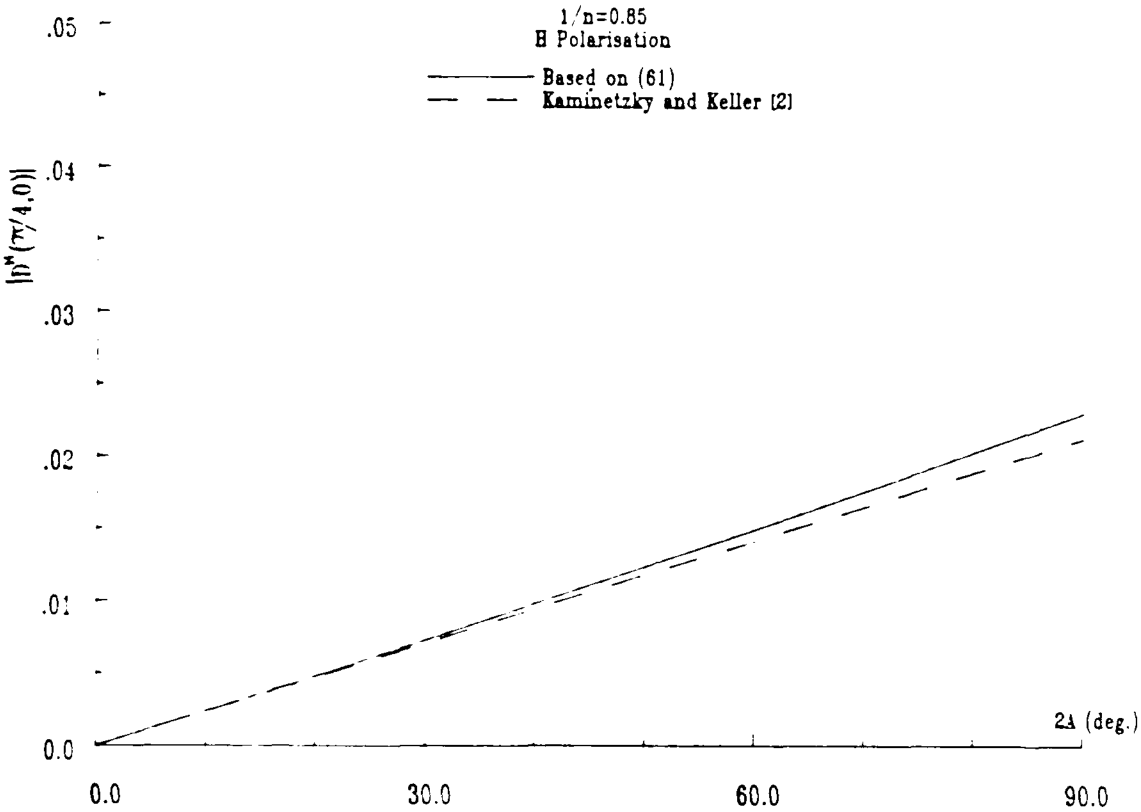

Figure 3(a). Magnitude of diffraction coefficient as a function of wedge angle for an H-polarised normally incident plane wave ($\theta = \pi/4$, $\theta_o = 0$).

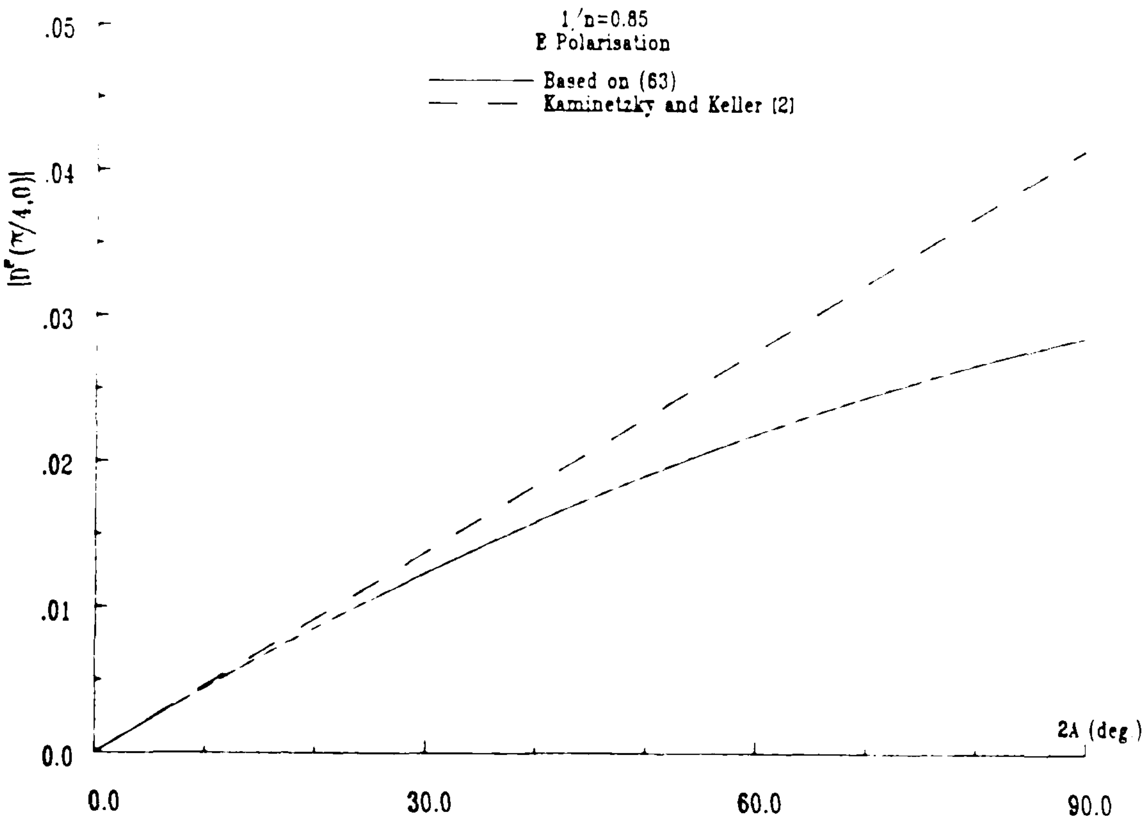

Figure 3(b). Magnitude of diffraction coefficient as a function of wedge angle for an E-polarised normally incident plane wave ($\theta = \pi/4$, $\theta_o = 0$).

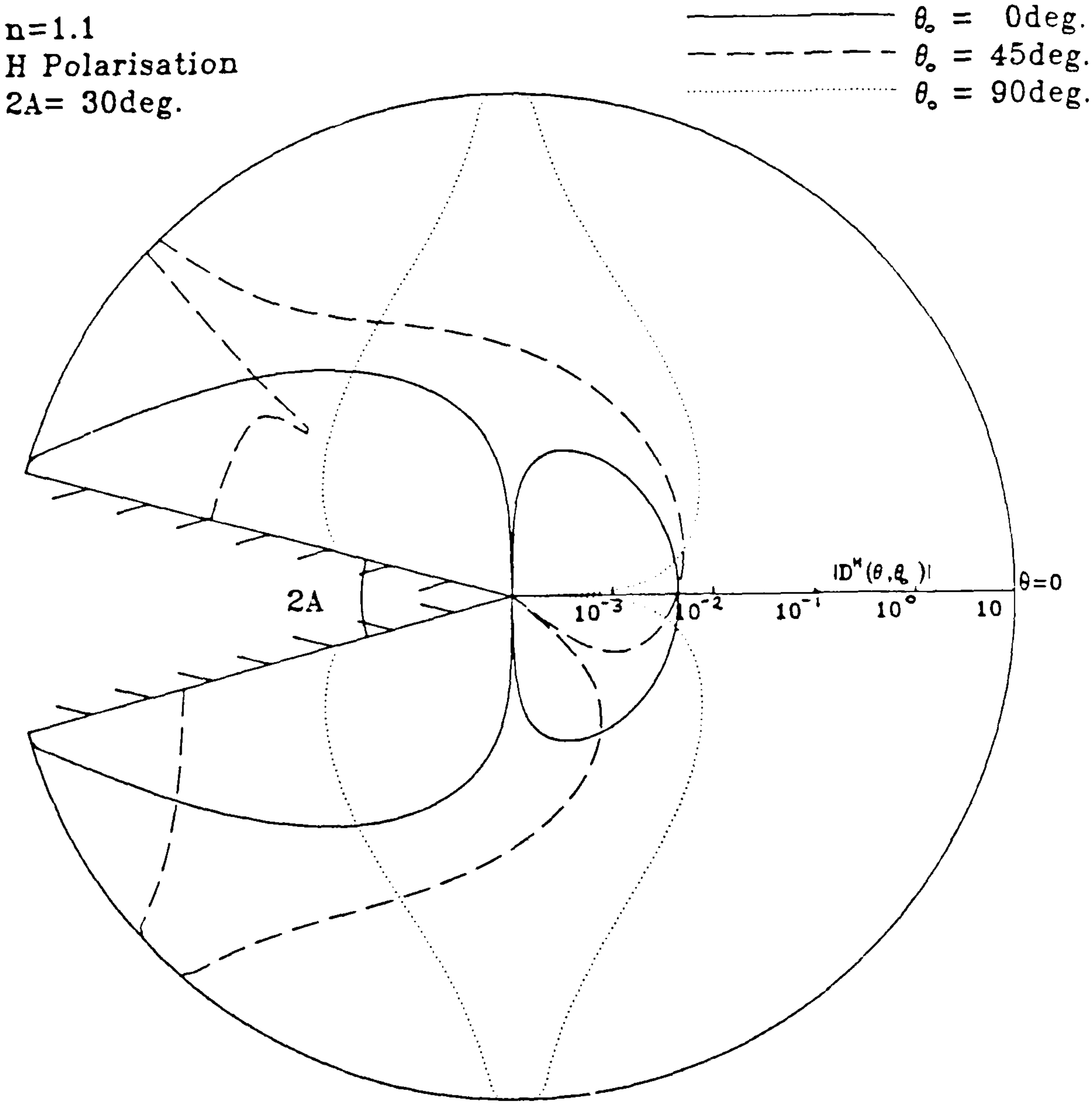

Figure 4(a). Magnitude of diffraction coefficient as a function of observation angle θ for an H-polarised normally incident plane wave.

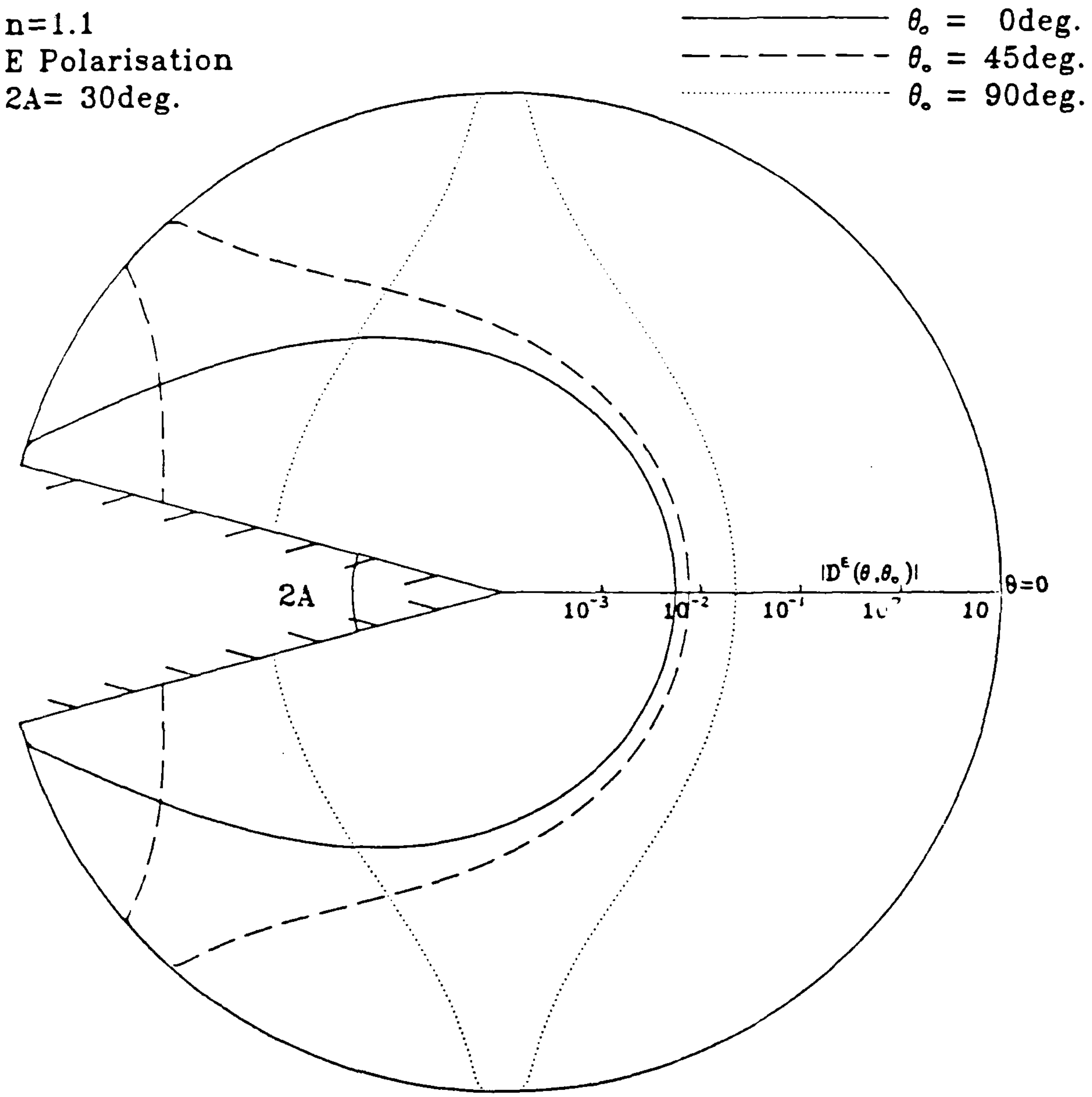

Figure 4(b). Magnitude of diffraction coefficient as a function of observation angle θ for an E-polarised normally incident plane wave.

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Abstract

The diffraction of a plane electromagnetic wave by a thin dielectric wedge is studied using a set of approximate boundary conditions together with the Wiener-Hopf technique. An integral expression for the scattered field is derived and the diffracted far field obtained by application of the method of steepest descents. The associated GTD diffraction coefficients are exact to order $O(A^2)$, where 2A is the (small) wedge angle. The results are useful for wedges for which $|n^2-1| A \ll 1$, where n is the refractive index of the dielectric relative to that of the surrounding medium.

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